Stereo vision with the use of a Virtual Plane in the space

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Abstract

This paper presents a geometrical method to calculate the position of points in 3D space from two different views. Our method is divided in two steps. The first step is two-dimensional and defines the epipolar geometry. It calculates a double projection of the homologous image on the reference image through a Virtual Intermediate Plane. We use eight corresponding points to calculate the transformation of the homologous image. The disparity between the corresponding points in a same referential allows us to get some information about the position of the 3D points. The second step calculates the coordinates of points in a projective space. The calculation of 3D coordinates consists in a simple transformation of projective to cartesian coordinates with 5 points in the case of a pinhole camera, and 4 points for the parallel projection. The interest of our method is to calculate the relative positionning without any knowledge on 3D coordinates of points, to offer controls along the calculation and to postpone use of the reference points at the end of the calculation.

Keywords: stereovision, geometry, calibration, relative positionning

1. Introduction

One major subject of research in computer vision is the stereovision i.e. the calculation of points in the space, using their images on different viewing positions by a pinhole camera. The conventional approach is based on camera calibration which implies to calculate the parameters of the cameras and their relative positioning. Some contribution to this approach can be found in [CHAU89] and [FAN_91]. This has been also the method of calculation used in photogrametry for many years [CARR71] [HURA60]. A recent approach was initiated by [LONG81] who presented a method to calculate the epipolar geometry that is infered by the pinhole model of camera. This work has been extended in 3D by algebraic methods that recover the camera transformation matrices with the use of five points of the scene [FAUG92] [LUON92], and with geometric methods that use two sets of four coplanar points in the scene [MORI93] [MOHR92]. Recently [SHAS93] has proposed a method of reconstruction using four points in the scene defining a tetrahedron and the center of projection of the camera as a fifth reference point. The author recovers a projective invariant and defines a new stuctural description: the projective depth.

Our approach differs from [SHAS93] in the way that we calculate the projective depth. We use a Virtual Intermediate Plane (VIP) which is defined by three points of the scene that are supposed to be known. We recover the projective depth by a double projection of a point on the homologous image on the reference one through the VIP.

The first step of our method is described in section 3 of this paper. It calculates the epipole and the transformation of the homologous image towards the reference one with the use of 8 corresponding points. The second step presented in section 4 calculates the relative positionning of the points in a projective referential made of 5 points supposed known. The cartesian coordinates are obtained at the end of the algorithm by a simple tranformation projective / Cartesian space, with the use of the coordinates of five points in the case of a pinhole model, and four points in the case of parallel projection.

The advantage of our method is that the first step gives controls on the validity of the corresponding points, and that the 2D transformation of the homologous image gives informations on the 3D position of points. This transformation is also useful for the global matching of the two images, because if one facet of the 3D object maps the VIP, then the corresponding parts in the images are directly matched.

2. General notations and preliminaries

The cartesian coordinates are noted in lower case and the projective coordinates in upper case. For all the points described below, we give the Cartesian and the projective coordinates.
We use two cameras S and S' and the homologous of an element x for S is noted x' on S'.

For the system S, the image plane is noted Q, the projection center: C (xc,yc,zc) or (Xc,Yc,Zc,Tc), the epipole: E (xe,ye) or (Xe,Ye,Ze), and the image points: Mi (xi,yi) or (Xi,Yi,Zi). The 3D points are noted Pi (xi*,yi*,zi*) or (Xi*,Yi*,Zi*,Ti*).

We also use a Virtual Plane in the space which is noted VIP. The points on the VIP are noted in lower case.
The two homographies corresponding to the projection of Q on VIP through C and the projection of Q' on VIP through C' are noted respectively H1 and H2.
The recalculation of a point Mi' from Q' to Q with the VIP method is noted Mi" (xi",yi") or (Xi",Yi",Zi").

We work on a pair of stereoscopic images. One is called the *reference image*. It remains unchanged during the calculation and corresponds to the plane Q of the system S. The other one is called the *homologous image*. It is transformed during the calculation and corresponds to the Q' plane of the system S'.

2.1 Epipolar geometry

The relation between the 3D points and their image on the projection plane is presented on Fig.1. We see that the 5 points P1,C,C',M1 and M1' belong to the same plane in the space and define the epipolar geometry [LONG81] which describes two pencils of lines that cross on the intersection of the two image planes. The homologous lines are called *epipolar lines* and the centers of the pencils the *epipoles*. The epipoles correspond to the intersection of the line defined by the projection centers with the image planes. This geometry leads to constraints that we will use in this paper.

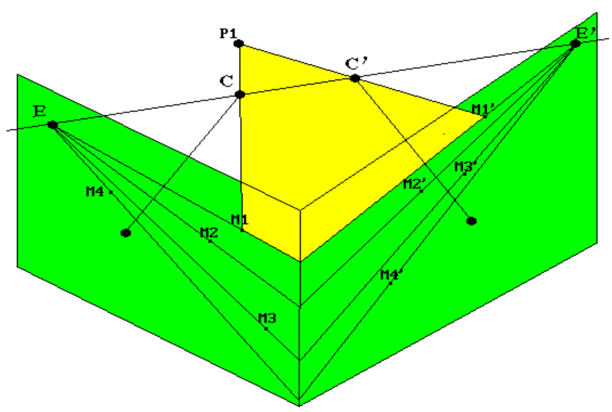


Fig.1: Epipolar geometry

2.2 Simplified stereovision

The fig.2 shows a simple case of stereo vision in which the two projection planes belong to the same plane defined by three points (not aligned). In this case there is only one epipole e.

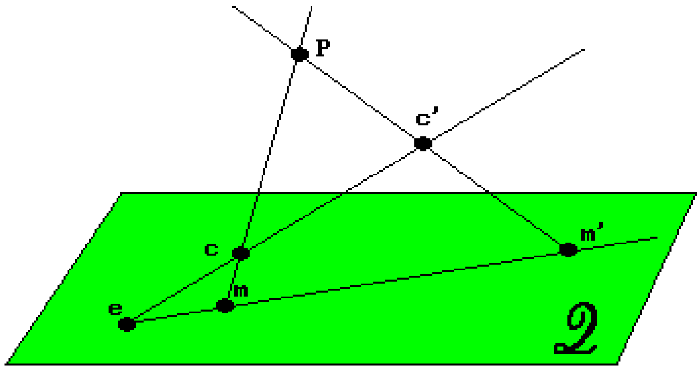


Fig.2: simplified stereovision

The calculation of the point P is easily done by crossing the lines {C,M} and {C',M'} in the space.

If the positions of C and C' are unknown, we can calculate them with the use of two points P1 and P2, and their images (M1,M1') (M2,M2') on Q. C is defined by the intersection of {P1,M1} and {P2,M2} and C' by the intersection of {P1,M1'} and {P2,M2'}. We have used three points to define Q and 2 points to calculate the projection centers.

Our goal is to bring the general case of stereovision to this particular case.

2.3 General case of stereovision

The usual problem of stereovision is presented on fig.3. The two projection planes are situated anywhere in the space but the pinhole model infers an epipolar geometry. All the points of the figure 3 belong to the same plane.

We bring this case of stereo vision to the particular case described in the beginning of this section by a method that will use a ***Virtual Intermediate Plane (V.I.P.)*** in the space.

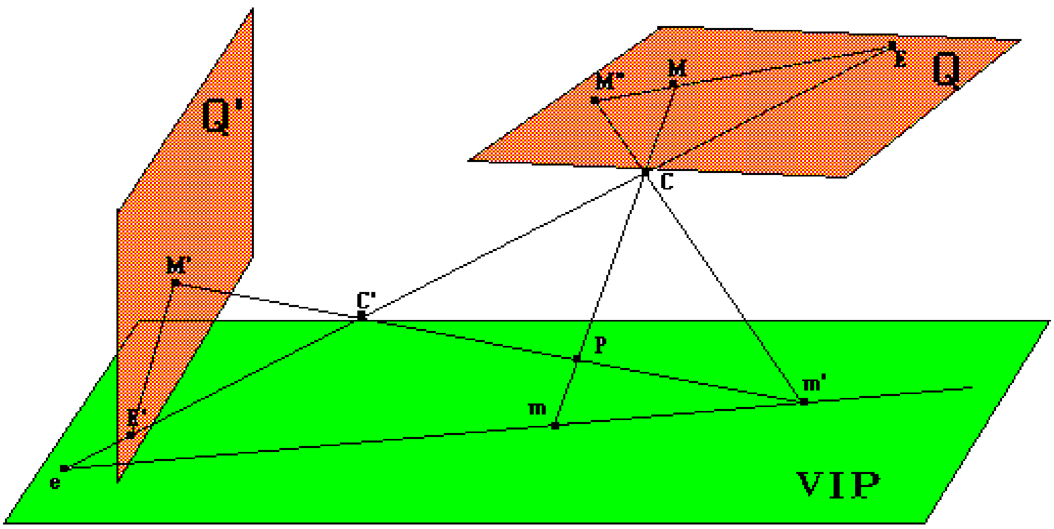


Fig.3: General case of stereovision

This method is divided in two steps:

The first one is two-dimensional. It defines the composition H of the projection H2 of Q' on the VIP through C', and the projection $H1^{-1}$ of the VIP on Q through C. If we consider the images of a 3D point P, the result of the calculation of its image M' on Q' is the point M'' on Q, which corresponds to the composition H of the two projections. The characteristics of the relation are that if the 3D point P belongs to the VIP, then the points M and M'' are fused, and if P does not belong to the VIP, then M and M'' are aligned with the epipole E on Q (epipolar geometry). The solution is calculated with 8 corresponding points on the two images.

The second step consists in projecting Q and H(Q') on the VIP, and calculating the position of C and C' in the space. Thus, we bring the general case of stereovision to the simplified case described at the beginning of this section. The calculation uses the coordinates of 5 non coplanar 3D points and their images. But as we can consider these 3D points as a projective referential, it is possible to calculate the coordinates in a projective space, and to obtain the relative positioning of all the points in the scene without the use of 3D coordinates.

3. Double projection and epipolar geometry

The projections H1 of Q on the VIP through C and the projection H2 of Q' on the VIP through C' are homographies from plane to plane.
Properties of the homographies [EFIM81]:
- a non degenerated homography is a bijective application defined by an inversible matrix.
- the homographies form a group, so a composition of homographies, or the inverse of an homography are also homographies.
- an homography preserves the cross ratio of four aligned points.

We calculate the composition H of the projections H2 and $H1^{-1}$ that transform the points Mi' on Q' to the points Mi'' on Q. This composition is an homography with the following properties:
1) The transformed images Mi''=H(Mi') of the points Pi that belong to the VIP map the points Mi on Q.
2) The transformed images Mi''=H(Mi') of the points Pi that are not on the VIP belong to the line {Mi,E} on Q.

3.1 Solution

We choose a projective referential on each plane made of three pairs of corresponding points (M1,M2,M3 / M1',M2',M3'), i.e. images of the same point in the space, and an arbitrary unity point, for example the center of gravity of each triplet of points. These triplets are called *basic points*. The 3D points corresponding to these basic points define the virtual plane in the space. The (xi,yi) Cartesian coordinates on Q are transformed into (Xi,Yi,Zi) projective coordinates by the matrix:

$$\begin{vmatrix} Xi \\ Yi \\ Zi \end{vmatrix} = \begin{vmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ 1 & 1 & 1 \end{vmatrix}^{-1} * \begin{vmatrix} xi \\ yi \\ 1 \end{vmatrix}$$

Tranformation into projective coordinates

We built a similar matrix for Q', with the coordinates of the homologous points.
So the points (M1,M2,M3) on Q and (M1',M2',M3') on Q' have the projective coordinates:

M1 (0,0,1),	M2 (0,1,0)	M3 (1,0,0)
M1'(0,0,1),	M2'(0,1,0)	M3'(1,0,0)

First property of H:

The transformed images Mi''=H(Mi') of the points Pi that belong to the VIP map the points Mi on Q.

H(M1') = M1, H(M2')=M2, H(M3')=M3

H(0,0,1)=(0,0,1), H(0,1,0)=(0,1,0), H(1,0,0)=(1,0,0)

Thus the homography is reduced to the matrix:

$$H = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$$

with a.b.c different from 0.

All the points on Q and Q' are transformed in projective coordinates with their respective matrix.

Second property of H:

The transformed images Mi''=H(Mi') of the points Pi that are not on the VIP belong to the line {Mi,E} on Q. These points are called *secondary points*. The alignment of the points {Mi,Mi',E} with the epipole (Xe,Ye,Ze) leads to the following system of equations:

$$\begin{vmatrix} Xi & a.Xi' & Xe \\ Yi & b.Yi' & Ye \\ Zi & c.Zi' & Ze \end{vmatrix} = 0$$

Equ.1: Alignment of homologous points with the epipole

Or:

$$Xe[c.Yi.Zi' - b.Zi.Yi'] - Ye[c.Xi.Zi' - a.Zi.Xi'] + Ze[b.Xi.Yi' - a.Yi.Xi'] = 0$$

We eliminate the coordinates of E, in order to obtain a better stability. For each equation, we take three secondary corresponding points with indices i,j,k (different from each other, and superior to 3 to avoid working with basic points that are fused by definition). The new system of equation is now:

$$\begin{vmatrix} c.Yi.Zi'-b.Yi'.Zi & a.Zi.Xi'-c.Zi'.Xi & b.Xi.Yi'-a.Xi'.Yi \\ c.Yj.Zj'-b.Yj'.Zj & a.Zj.Xj'-c.Zj'.Xj & b.Xj.Yj'-a.Xj'.Yj \\ c.Yk.Zk'-b.Yk'.Zk & a.Zk.Xk'-c.Zk'.Xk & b.Xk.Yk'-a.Xk'.Yk \end{vmatrix} = 0$$

Equ.2: System of equations to calculate the homography

With n points, we can build C(n,3) equations of this kind, that correspond to a system of equations with the bounded unknowns a, b, c:

$$A.b/c + A'.c/b + B.c/a + B'.a/c + C.a/b + C'.b/a + D = 0$$

Equ.3: Form of the equation to calculate the homography

In which the bounded unknowns can be written xa=b/c, xa'=c/b, xb=c/a, xb'=a/c, xc=a/b, xc'=b/a, so:

$$A.xa + A'.xa' + B.xb + B'.xb' + C.xc + C'.xc' + D = 0$$

Linear system of equation with six unknowns

With x1.x1' = x2.x2' = x3.x3' = 1

We find a least squares solution to this system of equation, and we obtain two controls of the solution:
- The least squares error LSQ of the system with C(n,3) equations and 8 unknowns.
- The error of coherence on the bounded variables which must respect the constraint x1.x1' = x2.x2' = x3.x3' = 1, noted ERRCOEF.

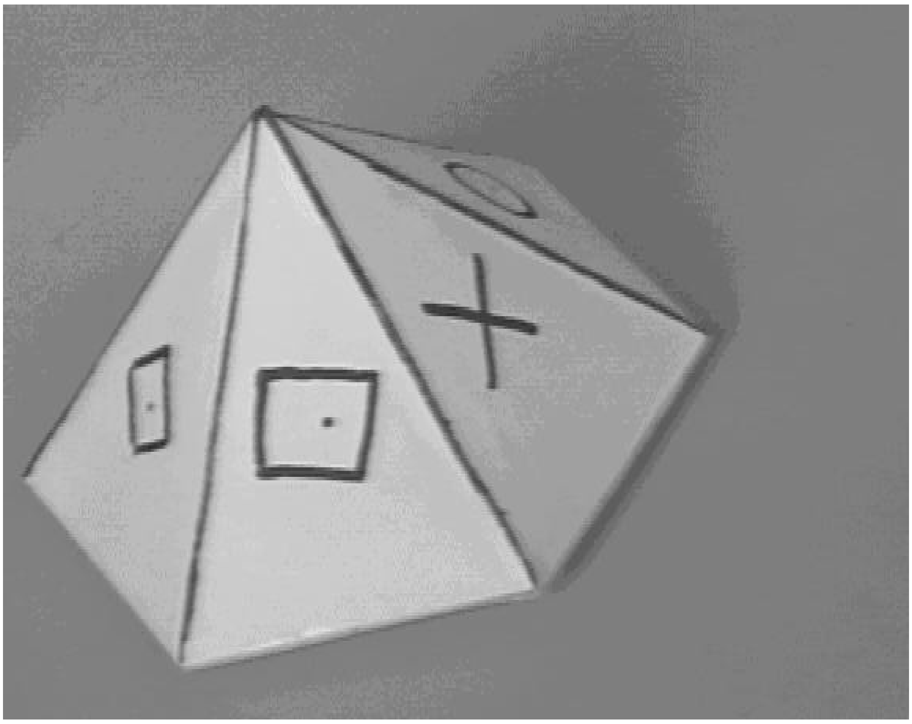
We calculate a solution with 8 corresponding points, 3 corresponding basic points that define the VIP and 5 corresponding secondary points that build C(5,3) = 10 linear equations

As the coefficients a,b,c are up to a factor, we can set c=1. We obtain the coefficients a, b and the controls described before. We can calculate the epipoles E and E'which have the same projective coordinates with the equation 1.

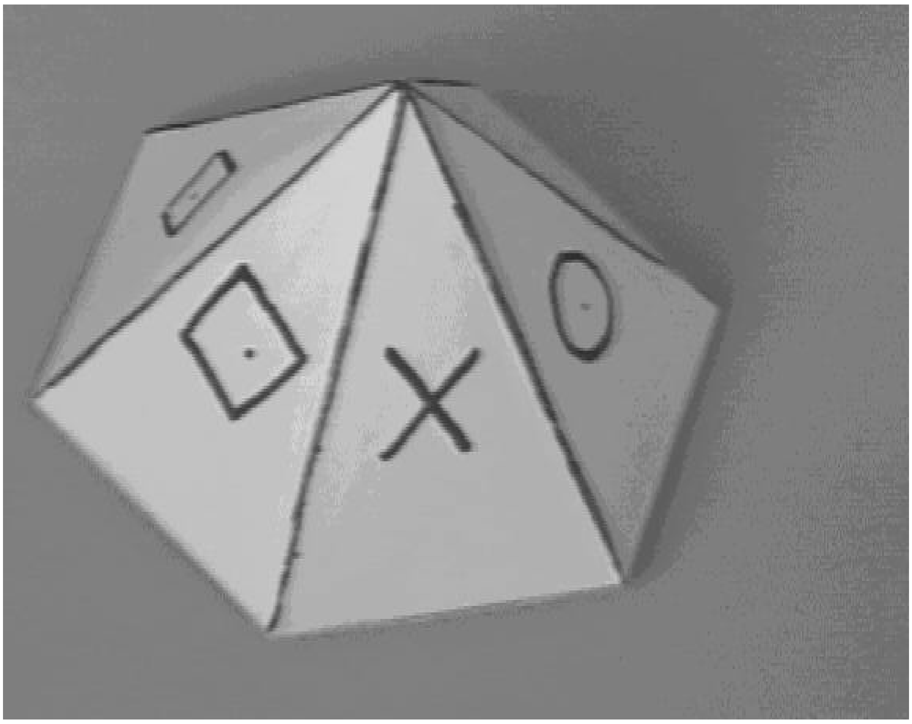
This solution allows us to recalculate the homologous image Q' and superpose it with the reference image Q. The corresponding basic points are fused and the corresponding secondary points aligned with the epipole. All the images Mi,Mi'' of the points Pi belonging to the VIP are fused, and the other aligned with the epipole. The homologous epipolar lines are superposed, so we have solved the problem of epipolar geometry. We do not need to know the position of the VIP in the space in this step.

3.2 Results

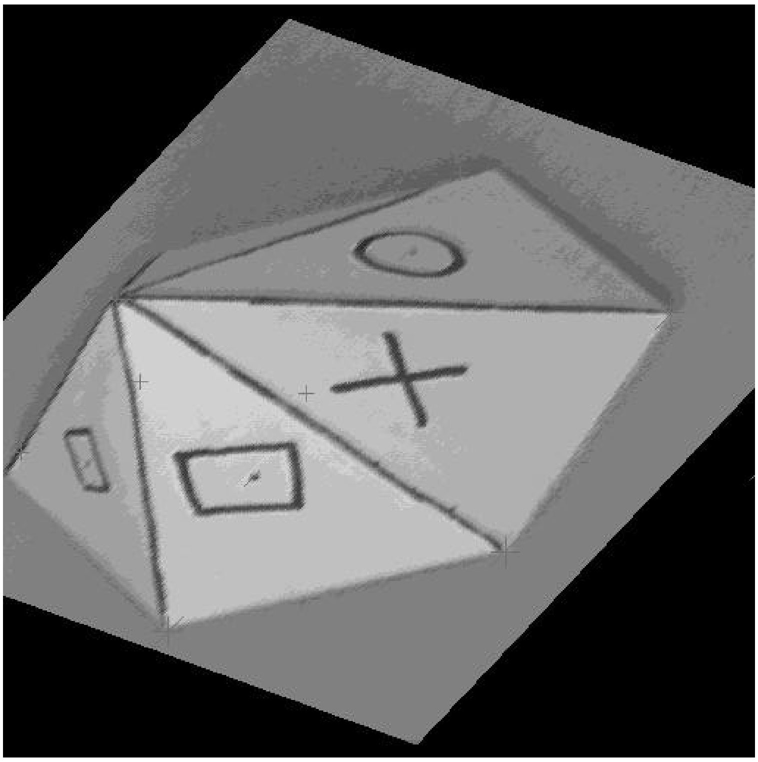
We have used our method on different kinds of images and we present here one example of solution with two images of a pyramid. The basic points (i.e. the VIP) are chosen on the base of the pyramid, and we can observe the transformation of the homologous image with the calculated homography. All the points of the basis of the pyramid are fused and the disparity between the other corresponding points is function of the distance of the 3D point from the VIP, and the distance from the camera. We also show the influence of the basic points on the transformation of the homologous image. In each case, the basic points form the vertices of the triangle drawn on the superposition of the reference image (which does not change) and the transformed homologous image.



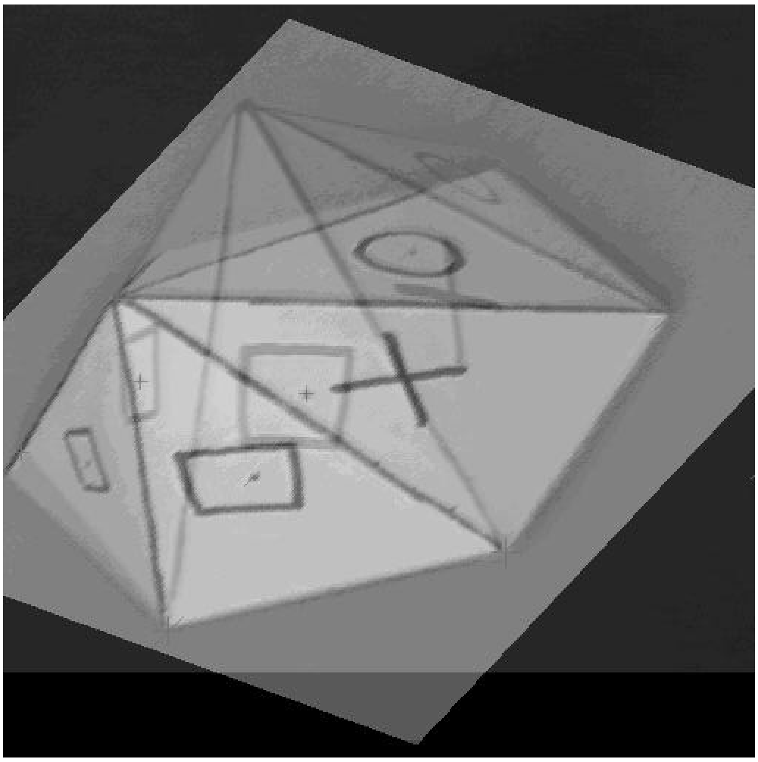
Reference image (1)



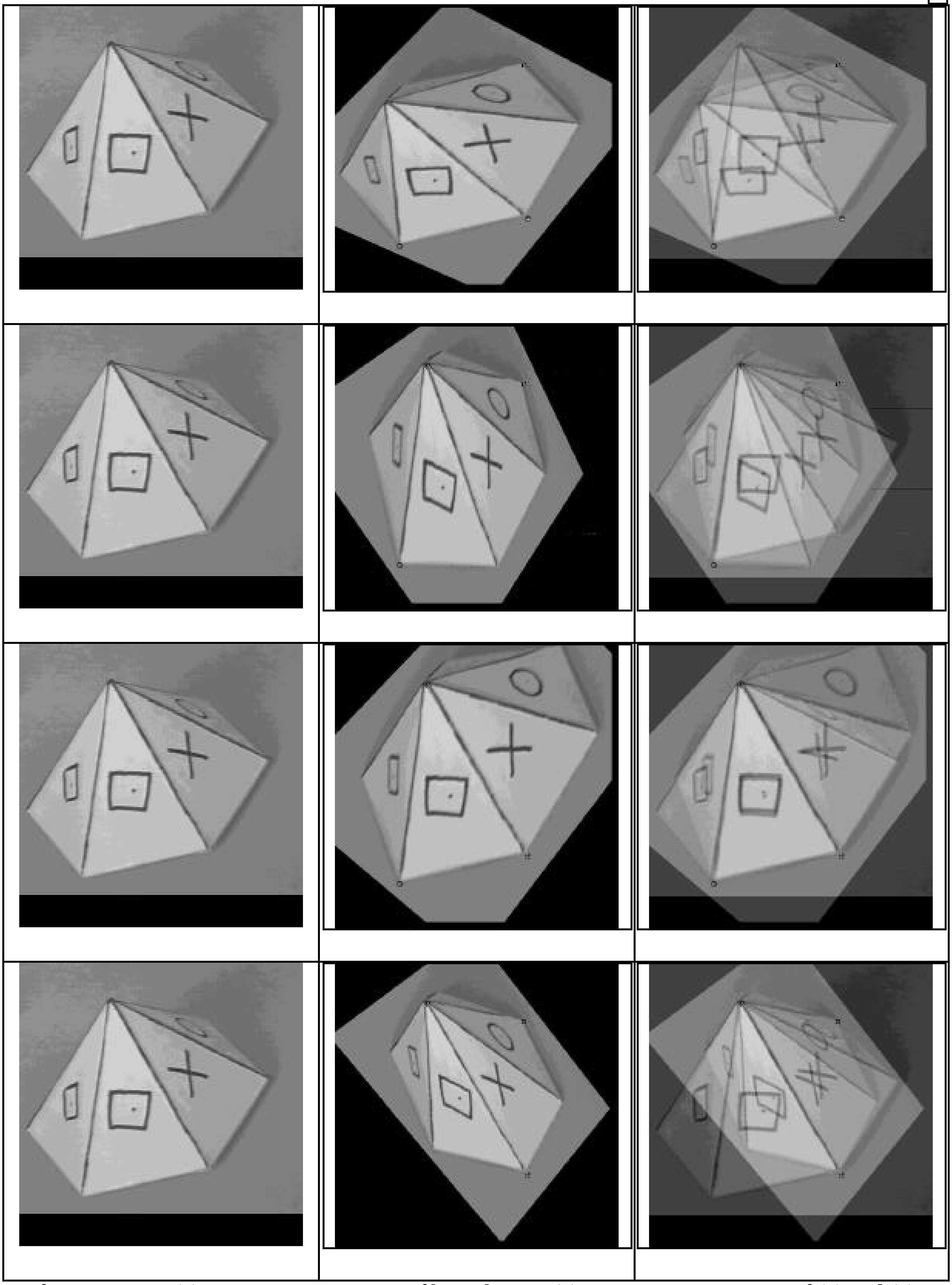
Homologous image (2)



Transformed homologous image (3)



Superposition of (1) and (3)



Reference image (1) Transf.homol.image(2) Superposition of (1) and (2)

Example of four different tranformations along with different basic points

4. Projection on the VIP and calculation of C and C'

4.1 Pinhole model

All the elements that we use for this second step of the processing are presented on fig. 4. We use the coordinates of five non coplanar points Pi in the space and their images Mi and Mi" on the image plane Q.

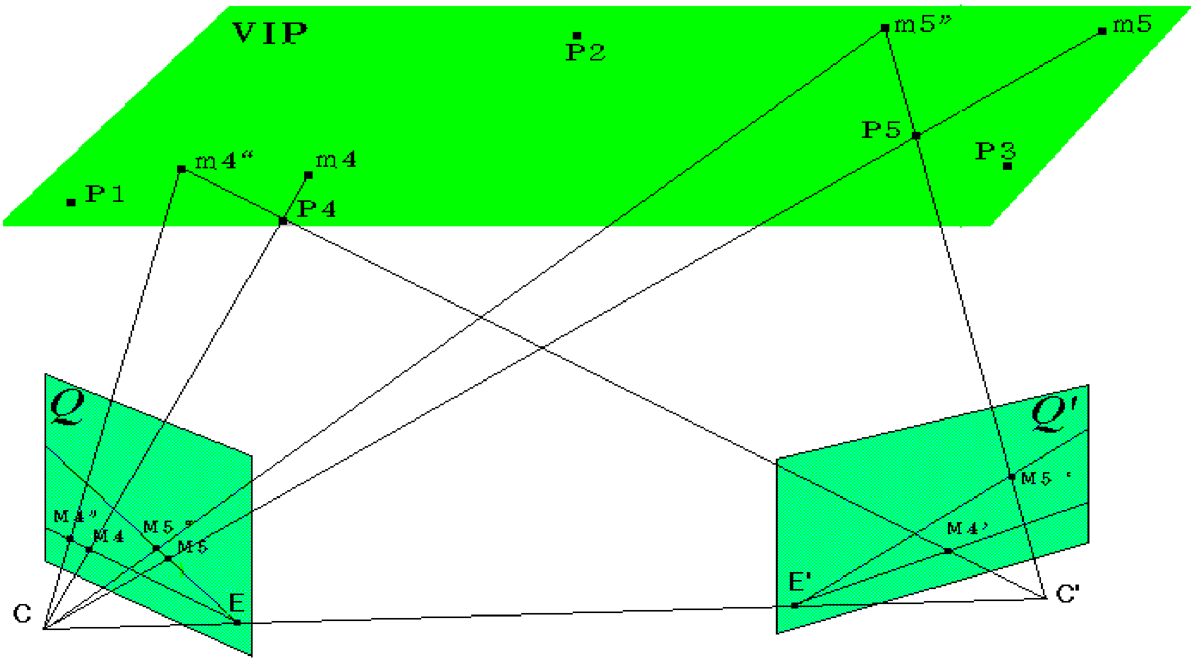


Fig.4: Alignment of points in the space

In order to bring the problem of stereovision to the simplified one presented in the section 2 of this paper, we have to project the image plane Q on the VIP and to calculate the coordinates of the projection centers C and C'.

We use the images {M1,M2,M3} and {M1',M2',M3'} of the 3D points {P1,P2,P3} as a projective referential on Q and Q' with the center of gravity of the three points as the unity point of each projective referential, like in the previous section. As we calculate the homography H between Q' and Q corresponding to the double projection through the VIP, we are able to transform the coordinates on Q' into H(Q') coordinates on Q. Thus we have only the plane Q to project through C on the VIP.

If we consider the points {P1,P2,P3,P4,P5} like a projective referential R of the space, then the projective coordinates of points are:
P1 (1,0,0,0), P2 (0,1,0,0) P3 (0,0,1,0) P4 (0,0,0,1) P5(1,1,1,1) and a point Pi in R has the coordinates Pi(Xi,Yi,Zi,Ti).

The VIP is defined by the three points (P1,P2,P3). Thus the equation of this plane is T=0 and the projection H1 of Q on the VIP is defined by an homography with two parameters u and v like for Q and Q'. We need also to calculate the coordinates of the projection centers that we can write C: (Xc,Yc,Zc,1) and C': (Xc',Yc',Zc',1). So we have a total of 8 unknown to calculate in order to define the projection of Q on the VIP and the projective coordinates of C and C' in the referential R.

The H1 projections of {M1,M2,M3,M4,M5} and {M1",M2",M3",M4",M5"} on the VIP have the coordinates m1=m1"(1,0,0,0), m2=m2"(0,1,0,0), m3=m3"(0,0,1,0), m4(u.x4,v.y4,z4,0), m4"(u.x4",v.y4",z4",0), m5(u.x5,v.y5,z5,0), m5"(u.x5",v.y5",z5",0) in R.

We consider the alignment of the points (m4,P4,C), (m4'',P4,C'), (m5,P5,C) and (m5'',P5,C'). Each line in the projective space R can be defined by the intersection of two planes, and leads to 2 equations.

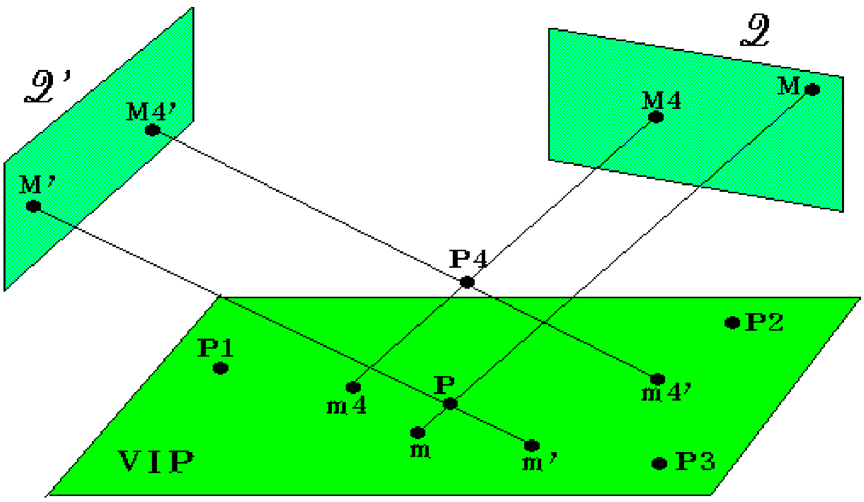
So we have a linear system of 8 equations with 8 unknown that gives us a unique solution, i.e. the projection H1 of Q through C on VIP and the projective coordinates of C and C'.

Now, we have asimple way to calculate the projective coordinates of an unknown 3D point P with the use of the projective coordinates of its two images M and M'' on Q. First we transform the projective coordinates M and M'' on Q into coordinates on the VIP m=H1(M), m''=H1(M''). Then we calculate the intersection of the lines (C,m) and (C',m'') in the projective space R. The result is the projective coordinates of P in R. The calculation of the cartesian coordinates is done with the application of the (4x4) projective -> cartesian matrix on the projective coordinates of P.

We have brought the general problem of stereo vision down to the simplified one presented in section 2. We are able to calculate the relative positionning of the points in a projective space if we don't know the 3D coordinates of the 5 points that form the projective referential, and we calculate the Cartesian coordinates if we know the position of these points in the space. The use of the 3D coordinates is postponed at the end of the calculation. This avoids the propagation of errors on the approximation of the reference points in the space.

4.2 Simplified solution with the parallel projection

The parallel projection is a simplified model of the pinhole model because the projection centers and the epipoles are situated at infinity. So we have only to calculate the directions of the projections on Q and Q'. Thus we need only four non coplanar points to solve the problem. The position of an unknown point P in the space is obtained by the intersection of the two parallels to (m4,P4) and (m4'',P4) passing respectively by m and m', as show on fig. 5.



Relations in the space with the parallel projection

In this particular case, we don't need to use the double projection, we simply convert the image coordinates into barycentric coordinates related to the referential {M1,M2,M3} for Q and {M1',M2',M3'} for Q' then we convert them to Cartesian coordinates related to the referential {P1,P2,P3} on the VIP.

$$\begin{vmatrix} x \\ y \\ 1 \end{vmatrix} = \begin{vmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ 1 & 1 & 1 \end{vmatrix} * \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}$$

Transformation into barycentric coordinates

Solution

1) Transformation of the coordinates into barycentric coordinates for the points Mi on Q with the referential {M1,M2,M3}, and for the points Mi' on Q' with the referential {M1',M2',M3'}.

2) Tranformation of the barycentric coordinates of the points Mi and Mi' into mi and mi' Cartesian coordinates on the VIP in relation with the referential {P1,P2,P3,P4}.
The 3D point P4 and its images on the VIP are known. So (m4,P4) and (m4',P4) define the two directions of the parallel projections.

3) Any point P is calculated by the intersection of the parallels to (m4,P4) and (m4',P4) passing respectively by m and m'.

4.3 Experimentation with the pyramid example

We have tested the two methods of calculation on several objects and we present here some results on the pyramid object. We have three kinds of errors on the evaluation of the points :
1) The camera is not a real pinhole model and there are some distortions on the image plane.
2) The position of the corresponding points on the images may have an error of one to three pixels from the real points.
3) The 3D points have been measured with a ruler and may have also some errors of approximation.

The aim of this experimentation is not to get the best accuracy but to evaluate the interest of our method for a normal use.
The depth of the object is about 10 cm and the distance camera - object is about one meter. The table 1 shows the positions of the points measured on the images (X,Y), (X',Y') and on the object (X3D,Y3D,Z3D).

X1	***Y1***	***X2***	***Y2***	***X3D***	***Y3D***	***Z3D***
15	*150*	*89*	*41*	*0*	*0*	*0*
454	*106*	*395*	*186*	*5*	*8.8*	*0*
343	*188*	*169*	*202*	*7.5*	*4.4*	*0*
164	*34*	*255*	*26*	*2.5*	*4.4*	*5*
93	*128*	*121*	*61*	*2.5*	*1.1*	*1.9*
306	*100*	*272*	*126*	*4.8*	*5.8*	*2.1*
336	*50*	*457*	*97*	*0*	*8.8*	*0*
112	*213*	*16*	*124*	*5*	*0*	*0*
206	*133*	*157*	*111*	*4.7*	*3.2*	*2*
312	*58*	*370*	*96*	*2.1*	*7.2*	*1.9*

Table 1: Coordordinates of the points on the images and on the object

The first step that calculates the double projection through the VIP gives the results:
Homography resulting from the two projections: a=0.997 b=0.986 c=1 LSQ=0 ERRCOEF=0.03
Position of the epipoles on Q and Q': E=(1860, -1177) E'=(1573, -922)
The least squares error LSQ and the error on the bounded variables ERRCOEF are small and show that the 2D solution is good. So we can apply the second part of our method of calculation.

The two next tables show the results of the 3D calculation with the pinhole model and the parallel projection. Each line of the table shows the position of one point (Xreal,Yreal,Zreal) measured on the object (corresponding to the table 1), the calculated point in 3D (Xcalc,Ycalc,Zcalc), the least squares error LSQ for the intersection of the two line in the space (4 equations and 3 variables) and the distance between the 3D measured point and the calculated one.

Pinhole model

Xréel	*Yréel*	*Zréel*	*Xcalc.*	*Ycalc.*	*Zcalc.*	*errmc*	*dist.*
-0.00	-0.00	-0.00	0.00	-0.00	0.00	0.00	0.00
5.00	8.80	0.00	5.00	8.80	0.00	0.00	0.00
7.50	4.40	0.00	7.50	4.40	0.00	0.00	0.00
2.10	7.20	1.90	2.10	7.20	1.90	0.00	0.00
2.50	4.40	5.00	2.50	4.40	5.00	0.00	0.00
2.50	1.10	1.90	2.52	1.53	1.82	18.21	0.44
4.80	5.80	2.10	4.90	5.70	2.17	17.08	0.16
0.00	8.80	-0.00	-0.94	8.72	-0.29	1.07	0.99
5.00	0.00	0.00	4.42	0.12	-0.48	3.61	0.76
4.70	3.20	2.00	5.00	3.17	2.07	41.42	0.31

Table 2: Results of the calculation with the pinhole model

Homography between the reference image Q and the VIP: u=0.0174 v=0.0183 w=0.017
Position of the projection centers: C=(50,5, -5.9, 37.9) C'=(79.9, 26.3, 80.7)
Total distance between the measured points and the calculated points: 2.6 cm, with represents a mean error of 2,6 mm.

Parallel projection

Xréel	*Yréel*	*Zréel*	*Xcalc.*	*Ycalc.*	*Zcalc.*	*errmc*	*dist.*
0.00	0.00	0.00	0.00	-0.00	0.00	0.00	0.00
5.00	8.80	0.00	5.00	8.80	0.00	0.00	0.00
7.50	4.40	0.00	7.50	4.40	0.00	0.00	0.00
2.10	7.20	1.90	2.10	7.20	1.90	0.00	0.00
2.50	4.40	5.00	1.80	4.32	5.10	0.03	0.71
2.50	1.10	1.90	2.08	1.38	1.80	0.02	0.51
4.80	5.80	2.10	4.52	5.72	2.16	0.00	0.30
0.00	8.80	0.00	0.00	8.54	-0.06	0.00	0.27
5.00	0.00	0.00	4.44	-0.14	-0.46	0.03	0.74
4.70	3.20	2.00	4.54	3.04	2.09	0.02	0.24

Table 3: Results of the 3D calculation with the parallel model

Total distance between the measured points and the calculated points: 2.7 cm. This corresponds to a mean error of 2,7 mm.

The tables 2 and 3 show the results of the two models of projection on the same pair of stereoscopic images. The reconstruction of the points is almost identical. This is due to the fact that if the object is compact enough and the distance of observation long enough, say more than 10 times the depth of the object, then the parallel model of projection is valid without too much distortion. The advantages are a better stability than the pinhole model, and also the use of only 4 points of the scene instead of 5 for the pinhole model.

The calculated position of the projection centers in the pinhole projection are irrelevant, although the reconstruction is acceptable. This poses the problem of the stability of the method of calculation which could be improved by the use of more than 5 points in the scene and models of deformation for the image planes.

5. Conclusion

The method of reconstruction of a 3D scene with the use of a Virtual Intermediate Plane presents several advantages:
- the calculation of epipolar geometry leads to controls on the validity of the corresponding points.
- the reconstruction of the 3D points can be done,knowing 5 non coplanar points in the case of the pinhole model, and 4 with the parallel projection. If no point of the scene is defined in 3D, the relative positionning can be calculated like in other similar methods [SHAS93] [FAUG92] [LUON92].
- The transformation of the homologous image in relation with the reference one is a first step towards the points matching. The advantage is to place the two images in the same referential, and to map the parts of the images corresponding to the VIP. This transformation can co-operate with form likelyhood criterion between contours in order to extract corresponding points on the two images. This algorithm will be presented in a future paper which is detailed in [COUA94].
- This method allows also the calculation in 2D of a facets model and presents the advantage of an easy calculation of disparities because all the corresponding points are mapped when the VIP corresponds to a real plane in the space.

The main goal to improve this method is to obtain a better stability by adding deformation models for the images planes and using more points in the scene.

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